Stability analysis of discontinuous steady states of a system consisting of one ODE and one reaction diffusion equation

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Assume a system of type

$$\begin{aligned} u_t &= f(u,w), && \text{on } \overline{\Omega} \\ w_t &= D\Delta w + g(u,w) && \text{in } \Omega, \\ (u(0),w(0)) &= (u_0,w_0) \in C^0(\overline{\Omega}) \times C^2(\Omega), \end{aligned} \tag{1}$$

with homogeneous Neumann boundary conditions for w and $f, g \in C^2$. We investigate stability of steady states $\tilde{u}, \tilde{w}$, where $\tilde{u}$ can have finitely many discontinuities. We show sufficient conditions under which a steady state with finitely many discontinuities is stable in a presented sense. Receptor based models proposed by Marciniak-Czochra [ref] are biologically motivated examples of this class of models. We show an example model resulting from a cell - bound receptor - free receptor model with Diffusion Driven Instability and stable discontinuous steady states.