Renormalization theory for transport equation

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Abstract

We consider transport equation $\partial_t u(t,x)+b(t,x)\nabla u(t,x)=0$. Characteristics method and Cauchy-Lipschitz theorem for ODE yield existence and uniqueness of the solutions to the transport equation with smooth vector field $b \in C^\infty(\mathbb{R}^{d+1})$. Following DiPerna and Lions we weaken the assumption on the vector field to $b \in W^{1,1}(\mathbb{R}^d) \cap L^\infty(\mathbb{R}^d)$ with $\mathrm{div}_x b \in L^\infty(\mathbb{R}^d)$ (in space). Under these assumptions using the renormalization theory we show existence and uniqueness of the solutions to the transport equation. In particular we first introduce the notion of renormalized solution for transport equation and then we use strong analytical tool - the so called commutator estimate - to prove the desired result.