Renormalised solutions of nonlinear parabolic problems with L^1 data

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Abstract

We study the behaviour of solutions u^ε regarding the parabolic equations of type:

$$\begin{cases} \frac{du^\varepsilon}{dt} - \mathrm{divA(t, x, Du^\varepsilon) = f^\varepsilon} & on \quad \Omega \times (0,T), \\ u^\varepsilon(t=0) = u_0^\varepsilon & \\ u^\varepsilon = 0 & on \quad \partial\Omega \times (0,T), \end{cases} \tag{1}$$

where u_0^ε and f^ε are smooth functions satisfying

- u_0^ε strongly converges in $L^1(\Omega)$,
- f^ε weakly converges in $L^1(\Omega \times (0,T,))$,

as ε tends to zero. We consider two types of operators $A(\cdot,\cdot,\cdot)$. In the first case we consider the identity for the last variable, which leads to the fact that (2) is standard heat equation. In the second case we speak about more general, nonlinear operator $A(\cdot,\cdot,\cdot)$. These results mostly follow from the Blanchard's and Murat's papers [1] and [2]. Additionally, I show some application of renormalised solutions regarding parabolic equations in real-life problems.

References

[1] Blanchard,D. *Truncations and monotonicity methods for parabolic equations*, Nonlinear Analysis: Theory, Methods and Applications, 1993, 21, 10, 725 - 743

[2] Blanchard,D. and Murat,F., *Renormalised solutions of nonlinear parabolic problems with L1 data: existence and uniqueness*, Proceedings of the Royal Society of Edinburgh: Section A Mathematics, 127, 06, 1137–1152