An application of the renormalized solutions of the steady continuity equation in the compressible Navier-Stokes equations

Ewelina Zatorska

Abstract

We shall list several results concerning the stationary continuity equation for compressible flow. These results mostly follow from the DiPerna-Lions theory developed for the transport equation [1]. In the second part of the talk we will investigate the weak compactness of the set of bounded energy renormalized weak solutions to system

$$\left.\begin{aligned} \operatorname{div} \varrho\mathbf{u} &= 0 \\ \operatorname{div}(\varrho\mathbf{u}\otimes\mathbf{u}) - \operatorname{div}\mathbb{S}(\mathbf{u}) + \nabla\varrho^\gamma &= \varrho\mathbf{f} \end{aligned}\right\} \quad \text{in } \Omega,$$

$$\Omega\in\mathbb{R}^3, \quad \mathbf{u}(x)=\mathbf{0},\ x\in\partial\Omega.$$

In particular, we will apply the DiPerna-Lions theory to explain Lions' proof of strong convergence of the density after the effective viscous flux identity [3]. We will also discuss Feireisl's method [2] developed for the case when ϱ is not a-priori bounded in $L^2(\Omega)$.

References

[1] R. J. DiPerna and P.-L. Lions. Ordinary differential equations, transport theory and Sobolev spaces. *Invent. Math.*, 98(3):511–547, 1989.

[2] E. Feireisl. On compactness of solutions to the compressible isentropic Navier-Stokes equations when the density is not square integrable. *Comment. Math. Univ. Carolin.*, 42(1):83–98, 2001.

[3] P.-L. Lions. *Mathematical topics in fluid mechanics. Vol. 2*, volume 10 of *Oxford Lecture Series in Mathematics and its Applications*. The Clarendon Press Oxford University Press, New York, 1998. Compressible models, Oxford Science Publications.