Delay Differential Equations in Population Biology and Epidemiology

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Abstract

In contrast to Ordinary Differential Equations (ODEs), Delay Differential Equations (DDEs) allow for the inclusion of past actions into mathematical models, thus making the model closer to the real-world phenomenon. Most of the results on DDEs have been developed only in the last fifty years (Kuang, 1993), however there is not yet such a comprehensive theory, as for ODEs. In particular, big challenges are given by equations with state-dependent delay. For this kind of problems, indeed, there is nowadays no unified theory, but rather results worked out on the basis of particular classes of equations (Hartung et al., 2006). Applications of DDEs with state-dependent delay is a very modern topic in mathematics and might offer the chance for significant steps forward.

In this talk we will present several applications of delay differential equations with constant and state-dependent delay in the fields of population dynamics and mathematical epidemiology:

- In the context of population dynamics, DDEs with constant delay can be obtained, e.g., from the balance laws of age-structured population dynamics, assuming that birth rates and death rates, as functions of age, are piece-wise constant. The delay arises naturally from biology as the age-at-maturity of individuals. We shall see applications of this modeling approach in population dynamics of isolated populations (Barbarossa et al., 2013), interplay of predators and prey (Mohr et al., 2013), tumor modeling (Barbarossa et al., 2012), as well as for the control of mosquitoes by mean of sterile insect technique.
- In epidemiology a delay can represent for example the duration in time of incubation or time a host stays infected. However, delay equations can be also used to investigate the phenomenon of waning immunity. When the body gets infected by a virus, indeed, the immune system develops a certain resistance against it. As a matter of fact disease-induced immunity tends to wane and, long time after recovery, an individual might become again susceptible to the virus. Exposure to the pathogen boosts the immune system, thus prolonging the time in which the individual is immune (Heffernan et al., 2009). We shall model the feedback mechanism which makes possible for certain individuals to have lifelong immunity, being regularly exposed to the infection.

References

- Y. Kuang, *Delay differential equations with applications in population dynamics* (1993).
- F. Hartung et al., *Functional differential equations with state-dependent delays: Theory and applications*, Handbook of Differential Equations 3 (2006).
- M. V. Barbarossa, C. Kuttler, J. Zinsl, *Delay equations modeling the effects of phase-specific drugs and immunotherapy on proliferating tumor cells*, Math. Biosc. Eng. 9(2) (2012).
- M. V. Barbarossa, K. P. Hadeler, C. Kuttler, *State-dependent Neutral Delay Equations from Population Models*, submitted for publication (2013).
- M. Mohr, M. V. Barbarossa, C. Kuttler, *Predator-prey interactions, age structures and delay equations*, in press (2014).
- J. M. Heffernan, M. J. Keeling, *Implications of vaccination and waning immunity*, Proc. R. Soc. B Biol. Sci. 276(1664) (2009).